13-1 Trigonometric Identities

Find the exact value of each expression if $0° < \theta < 90°$

1. If $\cot \theta = 2$, find $\tan \theta$.

SOLUTION:

$$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{2}$$

2. If $\sin\theta = \frac{4}{5}$, find $\cos \theta$.

SOLUTION:

$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \sin^2\theta$$
$$\cos^2\theta = 1 - \left(\frac{4}{5}\right)^2$$
$$\cos^2\theta = \frac{9}{25}$$
$$\cos\theta = \pm\frac{3}{5}$$

Since θ is in the first quadrant, $\cos\theta$ is positive.

Thus, $\cos\theta = \frac{3}{5}$.

3. If $\cos\theta = \frac{2}{3}$, find $\sin \theta$.

SOLUTION:

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta = 1 - \cos^2\theta$$
$$\sin^2\theta = 1 - \left(\frac{2}{3}\right)^2$$
$$\sin^2\theta = \frac{5}{9}$$
$$\sin\theta = \pm\frac{\sqrt{5}}{3}$$

Since θ is in the first quadrant, $\sin\theta$ is positive.

Thus, $\sin\theta = \frac{\sqrt{5}}{3}$.

4. If $\cos\theta = \frac{2}{3}$, find csc θ.

SOLUTION:

$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
\sin^2\theta &= 1 - \cos^2\theta \\
\left(\frac{1}{\csc\theta}\right)^2 &= 1 - \cos^2\theta \\
\left(\frac{1}{\csc\theta}\right)^2 &= 1 - \left(\frac{2}{3}\right)^2 \\
\left(\frac{1}{\csc\theta}\right)^2 &= \frac{5}{3} \\
\frac{1}{\csc\theta} &= \pm\frac{\sqrt{5}}{3} \\
\csc\theta &= \pm\frac{3}{\sqrt{5}}
\end{aligned}$$

Since θ is in the first quadrant, $\csc\theta$ is positive.

Thus, $\csc\theta = \frac{3}{\sqrt{5}}$.

Rationalize the denominator.

$$\begin{aligned}
\csc\theta &= \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\
&= \frac{3\sqrt{5}}{5}
\end{aligned}$$

Simplify each expression

5. tan θ cos^2 θ

SOLUTION:

$$\begin{aligned}
\tan\theta\cos^2\theta &= \frac{\sin\theta}{\cos\theta}\cos^2\theta \\
&= \sin\theta\cos\theta
\end{aligned}$$

6. csc^2 θ – cot^2 θ

SOLUTION:

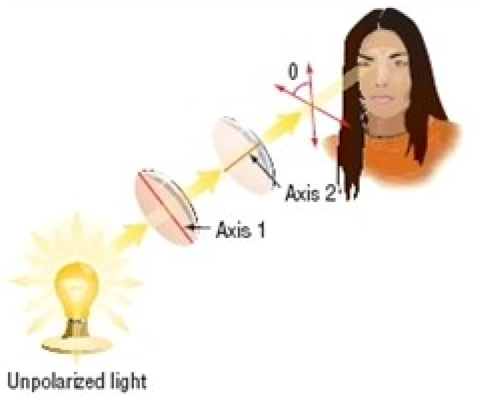
$$\begin{aligned}
\csc^2\theta - \cot^2\theta &= \left(\cot^2 + 1\right) - \cot^2\theta \\
&= \cot^2 + 1 - \cot^2\theta \\
&= 1
\end{aligned}$$

7. $\dfrac{\cos\theta\csc\theta}{\tan\theta}$

SOLUTION:

$$\begin{aligned}
\frac{\cos\theta\csc\theta}{\tan\theta} &= \frac{\cos\theta\dfrac{1}{\sin\theta}}{\tan\theta} \\
&= \frac{\cos\theta}{\sin\theta} \div \tan\theta \\
&= \frac{\cos\theta}{\sin\theta} \div \frac{\sin\theta}{\cos\theta} \\
&= \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\sin\theta} \\
&= \left(\frac{\cos\theta}{\sin\theta}\right)^2 \\
&= \cot^2\theta
\end{aligned}$$

8. **CCSS PERSEVERANCE** When unpolarized light passes through polarized sunglass lenses, the intensity of the light is cut in half. If the light then passes through another polarized lens with its axis at an angle of θ to the first, the intensity of the emerging light can be found by using the formula $I = I_o - \frac{I_o}{\csc^2 \theta'}$, where I_o is the intensity of the light incoming to the second polarized lens, I is the intensity of the emerging light, and θ is the angle between the axes of polarization.
a. Simplify the formula in terms of cos θ.
b. Use the simplified formula to determine the intensity of light that passes through a second polarizing lens with axis at 30° to the original.



SOLUTION:

a.

$$\begin{aligned} I &= I_o - \frac{I_o}{\csc^2 \theta'} \\ &= I_o\left(1 - \frac{1}{\csc^2 \theta'}\right) \\ &= I_o\left(1 - \left(\frac{1}{\csc \theta}\right)^2\right) \\ &= I_o\left(1 - \sin^2 \theta\right) \\ &= I_o \cos^2 \theta \end{aligned}$$

b.Substitute 30° for θ.

$$\begin{aligned} I_o \cos^2 \theta &= I_o \cos^2 30° \\ &= I_o\left(\frac{\sqrt{3}}{2}\right)^2 \\ &= \frac{3}{4} I_o \end{aligned}$$

The light has three-fourths the intensity it had before passing through the second polarizing lens.

Find the exact value of each expression $0° < \theta < 90°$

9. If $\cos\theta = \frac{3}{5}$, find csc θ.

SOLUTION:

$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
\sin^2\theta &= 1 - \cos^2\theta \\
\left(\frac{1}{\csc\theta}\right)^2 &= 1 - \cos^2\theta \\
\left(\frac{1}{\csc\theta}\right)^2 &= 1 - \left(\frac{3}{5}\right)^2 \\
\left(\frac{1}{\csc\theta}\right)^2 &= \frac{16}{25} \\
\frac{1}{\csc\theta} &= \pm\frac{4}{5} \\
\csc\theta &= \pm\frac{5}{4}
\end{aligned}$$

Since θ is in the first quadrant, $\csc\theta$ is positive.

Thus, $\csc\theta = \frac{5}{4}$.

10. If $\sin\theta = \frac{1}{2}$, find tan *θ*.

SOLUTION:

$$\sin^2\theta + \cos^2\theta = 1$$
$$\left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{3}{4}$$
$$\cos\theta = \pm\frac{\sqrt{3}}{2}$$

Since θ is in the first quadrant, $\cos\theta$ is positive.

Thus, $\cos\theta = \frac{\sqrt{3}}{2}$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$= \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$
$$= \frac{1}{\sqrt{3}}$$
$$= \frac{\sqrt{3}}{3}$$

11. If $\sin\theta = \frac{3}{5}$, find cos *θ*.

SOLUTION:

$$\sin^2\theta + \cos^2\theta = 1$$
$$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{16}{25}$$
$$\cos\theta = \pm\frac{4}{5}$$

Since θ is in the first quadrant, $\cos\theta$ is positive.

Thus, $\cos\theta = \frac{4}{5}$.

12. If tan θ = 2, find sec θ.

SOLUTION:

$$\begin{aligned} 1+\tan^2\theta &= \sec^2\theta \\ 1+2^2 &= \sec^2\theta \\ \sec^2\theta &= 5 \\ \sec\theta &= \sqrt{5} \end{aligned}$$

Since θ is in the first quadrant, $\sec\theta$ is positive.
Thus, $\sec\theta = \sqrt{5}$.

Find the exact value of each expression 180° < θ < 270°

13. If $\cos\theta = -\frac{3}{5}$, find csc θ.

SOLUTION:

$$\begin{aligned} \sin^2\theta+\cos^2\theta &= 1 \\ \frac{1}{\csc^2\theta}+\left(-\frac{3}{5}\right)^2 &= 1 \\ \frac{1}{\csc^2\theta} &= \frac{16}{25} \\ \csc^2\theta &= \frac{25}{16} \\ \csc\theta &= \pm\frac{5}{4} \end{aligned}$$

Since θ is in the third quadrant, $\csc\theta$ is negative.
Therefore, $\csc\theta = -\frac{5}{4}$.

14. If $\sec\theta = -3$, find tan θ.

SOLUTION:

$$\begin{aligned} 1+\tan^2\theta &= \sec^2\theta \\ 1+\tan^2\theta &= (-3)^2 \\ \tan^2\theta &= 8 \\ \tan\theta &= \pm 2\sqrt{2} \end{aligned}$$

Since θ is in the third quadrant, $\tan\theta$ is positive.
Therefore, $\tan\theta = 2\sqrt{2}$.

15. If $\cot\theta = \frac{1}{4}$, find csc θ.

SOLUTION:

$$1 + \cot^2\theta = \csc^2\theta$$
$$1 + \left(\frac{1}{4}\right)^2 = \csc^2\theta$$
$$\csc^2\theta = \frac{17}{16}$$
$$\csc\theta = \pm\frac{\sqrt{17}}{4}$$

Since θ is in the third quadrant, $\csc\theta$ is negative.

Therefore, $\csc\theta = -\frac{\sqrt{17}}{4}$.

16. If $\sin\theta = -\frac{1}{2}$, find cos θ.

SOLUTION:

$$\sin^2\theta + \cos^2\theta = 1$$
$$\left(-\frac{1}{2}\right)^2 + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{3}{4}$$
$$\cos\theta = \pm\frac{\sqrt{3}}{2}$$

Since θ is in the third quadrant, $\cos\theta$ is negative.

Therefore, $\cos\theta = -\frac{\sqrt{3}}{2}$.

13-1 Trigonometric Identities

Find the exact value of each expression 270° < θ < 360°

17. If $\cos\theta = \frac{5}{13}$, find sin θ.

SOLUTION:

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta + \left(\frac{5}{13}\right)^2 = 1$$
$$\sin^2\theta = \frac{144}{169}$$
$$\sin\theta = \pm\frac{12}{13}$$

Since θ is in the fourth quadrant, $\sin\theta$ is negative.

Therefore, $\sin\theta = -\frac{12}{13}$.

18. If $\tan\theta = -1$, find sec θ.

SOLUTION:

$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + (-1)^2 = \sec^2\theta$$
$$\sec^2\theta = 2$$
$$\sec\theta = \pm\sqrt{2}$$

Since θ is in the fourth quadrant, $\sec\theta$ is positive.
Therefore, $\sec\theta = \sqrt{2}$.

19. If $\sec\theta = \frac{5}{3}$, find cos θ.

SOLUTION:

$$\sec\theta = \frac{5}{3}$$
$$\frac{1}{\cos\theta} = \frac{5}{3}$$
$$\cos\theta = \frac{3}{5}$$

13-1 Trigonometric Identities

20. If $\csc\theta = -\frac{5}{3}$, find cos θ.

SOLUTION:

$$\csc\theta = -\frac{5}{3}$$
$$\sin\theta = -\frac{3}{5}$$

$$\begin{aligned}\sin^2\theta + \cos^2\theta &= 1\\ \left(-\frac{3}{5}\right)^2 + \cos^2\theta &= 1\\ \cos^2\theta &= \frac{16}{25}\\ \cos\theta &= \pm\frac{4}{5}\end{aligned}$$

Since θ is in the fourth quadrant, $\cos\theta$ is positive.

Therefore, $\cos\theta = \frac{4}{5}$.

Simplify each expression

21. $\sec\theta\tan^2\theta + \sec\theta$

SOLUTION:

$$\begin{aligned}\sec\theta\tan^2\theta + \sec\theta &= \sec\theta\left(\tan^2\theta + 1\right)\\ &= \sec\theta\left(\sec^2\theta\right)\\ &= \sec^3\theta\end{aligned}$$

22. $\cos\left(\frac{\pi}{2} - \theta\right)\cot\theta$

SOLUTION:

$$\begin{aligned}\cos\left(\frac{\pi}{2} - \theta\right)\cot\theta &= \sin\theta\cdot\frac{\cos\theta}{\sin\theta}\\ &= \cos\theta\end{aligned}$$

23. $\cot\theta\sec\theta$

SOLUTION:

$$\begin{aligned}\cot\theta\sec\theta &= \frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\cos\theta}\\ &= \frac{1}{\sin\theta}\\ &= \csc\theta\end{aligned}$$

13-1 Trigonometric Identities

24. $\sin\theta(1+\cot^2\theta)$

SOLUTION:

$$\begin{aligned}\sin\theta\left(1+\cot^2\theta\right) &= \sin\theta\left(\csc^2\theta\right)\\ &= \sin\theta\left(\frac{1}{\sin^2\theta}\right)\\ &= \frac{1}{\sin\theta}\\ &= \csc\theta\end{aligned}$$

25. $\sin\left(\frac{\pi}{2}-\theta\right)\sec\theta$

SOLUTION:

$$\begin{aligned}\sin\left(\frac{\pi}{2}-\theta\right)\sec\theta &= \cos\theta\left(\sec\theta\right)\\ &= \cos\theta\cdot\frac{1}{\cos\theta}\\ &= 1\end{aligned}$$

26. $\frac{\cos(-\theta)}{\sin(-\theta)}$

SOLUTION:

$$\begin{aligned}\frac{\cos\left(-\theta\right)}{\sin\left(-\theta\right)} &= \frac{\cos\theta}{-\sin\theta}\\ &= -\cot\theta\end{aligned}$$

27. **ELECTRONICS** When there is a current in a wire in a magnetic field, such as in a hairdryer, a force acts on the wire. The strength of the magnetic field can be determined using the formula $B=\frac{F\csc\theta}{I\ell}$, where F is the force on the wire, I is the current in the wire, ℓ is the length of the wire, and θ is the angle the wire makes with the magnetic field. Rewrite the equation in terms of sin θ (Hint : Solve for F.)

SOLUTION:

$$\begin{aligned}B &= \frac{F\csc\theta}{I\ell}\\ F\csc\theta &= I\ell B\\ F &= \frac{BI\ell}{\csc\theta}\\ F &= BI\ell\sin\theta\end{aligned}$$

Simplify each expression.

28. $\dfrac{1-\sin^2\theta}{\sin^2\theta}$

SOLUTION:

$$\begin{aligned}\frac{1-\sin^2\theta}{\sin^2\theta} &= \frac{1}{\sin^2\theta}-\frac{\sin^2\theta}{\sin^2\theta}\\ &= \csc^2\theta-1\\ &= \cot^2\theta\end{aligned}$$

29. $\tan\theta\csc\theta$

SOLUTION:

$$\begin{aligned}\tan\theta\csc\theta &= \frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\sin\theta}\\ &= \frac{1}{\cos\theta}\\ &= \sec\theta\end{aligned}$$

30. $\dfrac{1}{\sin^2\theta}-\dfrac{\cos^2\theta}{\sin^2\theta}$

SOLUTION:

$$\begin{aligned}\frac{1}{\sin^2\theta}-\frac{\cos^2\theta}{\sin^2\theta} &= \csc^2\theta-\cot^2\theta\\ &= 1\end{aligned}$$

31. $2(\csc^2\theta-\cot^2\theta)$

SOLUTION:

$$\begin{aligned}2\left(\csc^2\theta-\cot^2\theta\right) &= 2(1)\\ &= 2\end{aligned}$$

32. $(1+\sin\theta)(1-\sin\theta)$

SOLUTION:

$$\begin{aligned}(1+\sin\theta)(1-\sin\theta) &= 1-\sin^2\theta\\ &= \cos^2\theta\end{aligned}$$

33. $2-2\sin^2\theta$

SOLUTION:

$$\begin{aligned}2-2\sin^2\theta &= 2(1-\sin^2\theta)\\ &= 2\cos^2\theta\end{aligned}$$

34. **SUN** The ability of an object to absorb energy is related to a factor called the emissivity e of the object. The emissivity can be calculated by using the formula $e = \frac{W \sec\theta}{AS}$, where W is the rate at which a person's skin absorbs energy from the Sun, S is the energy from the Sun in watts per square meter, A is the surface area exposed to the Sun, and θ is the angle between the Sun's rays and a line perpendicular to the body.

a. Solve the equation for W. Write your answer using only sin θ or cos θ.

b. Find W if $e = 0.80$, $\theta = 40°$, $A = 0.75\text{m}^2$, and $S = 1000$ W/m2. Round to the nearest hundredth.

SOLUTION:

a.

$$\begin{aligned} e &= \frac{W \sec\theta}{AS} \\ ASe &= W \sec\theta \\ W &= \frac{ASe}{\sec\theta} \\ &= ASe\cos\theta \end{aligned}$$

b. Substitute the values of e, θ, A and S and evaluate.

$$\begin{aligned} W &= ASe\cos\theta \\ &= 0.75(1000)(0.80)\cos 40° \\ &\approx 459.63 \end{aligned}$$

35. **CCSS MODELING** The map shows some of the buildings in Maria's neighborhood that she visits on a regular basis. The sine of the angle θ formed by the roads connecting the dance studio, the school, and Maria's house is $\frac{4}{9}$.

a. What is the cosine of the angle?

b. What is the tangent of the angle?

c. What are the sine, cosine, and tangent of the angle formed by the roads connecting the piano teacher's house, the school, and Maria's house?

SOLUTION:

a. Given $\sin\theta = \frac{4}{9}$.

$$\sin^2\theta + \cos^2\theta = 1$$
$$\left(\frac{4}{9}\right)^2 + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \frac{16}{81}$$
$$\cos\theta = \frac{\sqrt{65}}{9}$$

b. Given $\sin\theta = \frac{4}{9}$ and $\cos\theta = \frac{\sqrt{65}}{9}$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{4}{9}}{\frac{\sqrt{65}}{9}} = \frac{4}{\sqrt{65}} = \frac{4\sqrt{65}}{65}$$

c.

$$\sin(180-\theta) = \sin\theta = \frac{4}{9}$$
$$\cos(180-\theta) = -\cos\theta = -\frac{\sqrt{65}}{9}$$
$$\tan(180-\theta) = -\tan\theta = -\frac{4\sqrt{65}}{65}$$

36. **MULTIPLE REPRESENTATIONS** In this problem, you will use a graphing calculator to determine whether an equation may be a trigonometric identity. Consider the trigonometric identity $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$.
a. TABULAR Complete the table.

θ	0°	30°	45°	60°
$\tan^2\theta - \sin^2$				
$\tan^2\theta\sin^2\theta$				

b. GRAPHICAL Use a graphing calculator to graph $\tan^2\theta - \sin^2\theta = \tan^2\theta\ \sin^2\theta$ as two separate functions. Sketch the graph.

c. ANALYTICAL If the graphs of the two functions do not match, then the equation is not an identity. Do the graphs coincide?

d. ANALYTICAL Use a graphing calculator to determine whether the equation $\sec^2 x - 1 = \sin^2 x\ \sec^2 x$ may be an identity. (Be sure your calculator is in degree mode.)

SOLUTION:

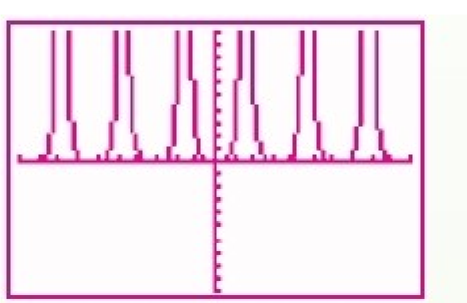
a.

$$\tan^2 0° = 0$$
$$\sin^2 0° = 0$$
$$\tan^2 30° = \left(\frac{\sqrt{3}}{3}\right)^2 = \frac{1}{3}$$
$$\sin^2 30° = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$
$$\tan^2 45° = (1)^2 = 1$$
$$\sin^2 45° = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$$
$$\tan^2 60° = \left(\sqrt{3}\right)^2 = 3$$
$$\sin^2 60° = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

θ	0°	30°	45°	60°
$\tan^2\theta - \sin^2$	0	$\frac{1}{12}$	$\frac{1}{2}$	$\frac{9}{4}$
$\tan^2\theta\sin^2\theta$	0	$\frac{1}{12}$	$\frac{1}{2}$	$\frac{9}{4}$

b. KEYSTROKES:

Y= TAN ALPHA [x]) x^2 – SIN ALPHA [x]) x^2 ENTER TAN ALPHA [x]) x^2 SIN ALPHA [x]) x^2 GRAPH

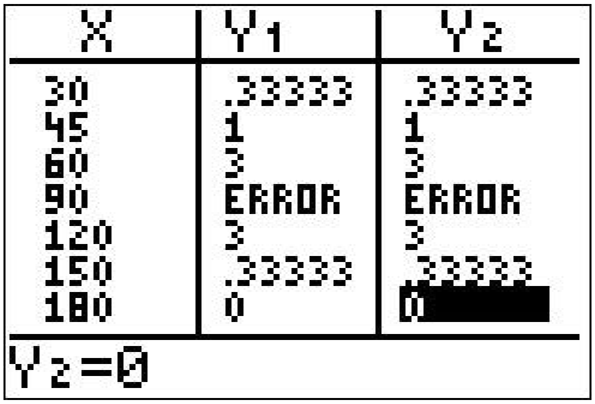


[–540, 540] scl: 90 by [–10, 10] scl: 1

c. yes

d.

Plug in $\sec^2 x - 1$ for Y1 and $\sin^2 x \sec^2 x$ for Y2 in a graphing calculator and form a table.



X	Y1	Y2
30	.33333	.33333
45	1	1
60	3	3
90	ERROR	ERROR
120	3	3
150	.33333	.33333
180	0	0

Y2=0

From the table, the values of $\sec^2 x - 1$ and $\sin^2 x \sec^2 x$ are the same. Therefore, the equation $\sec^2 x - 1 = \sin^2 x \sec^2 x$ is an identity.

37. **SKIING** A skier of mass m descends a θ-degree hill at a constant speed. When Newton's laws are applied to the situation, the following system of equations is produced: $F_n - mg\cos\theta = 0$ and $mg\sin\theta - \mu_k F_n = 0$, where g is the acceleration due to gravity, F_n is the normal force exerted on the skier, and μ_k is the coefficient of friction. Use the system to define μ_k as a function of θ.

SOLUTION:

$$\begin{aligned} F_n - mg\cos\theta &= 0 \\ F_n &= mg\cos\theta \end{aligned}$$

Substitute $mg\cos\theta$ for F_n and solve for μ_k.

$$\begin{aligned} mg\sin\theta - \mu_k F_n &= 0 \\ \mu_k F_n &= mg\sin\theta \\ \mu_k mg\cos\theta &= mg\sin\theta \\ \mu_k &= \frac{mg\sin\theta}{mg\cos\theta} \\ &= \tan\theta \end{aligned}$$

Simplify each Expression.

38. $\dfrac{\tan\left(\dfrac{\pi}{2}-\theta\right)\sec\theta}{1-\csc^2\theta}$

SOLUTION:

$$\begin{aligned}\frac{\tan\left(\dfrac{\pi}{2}-\theta\right)\sec\theta}{1-\csc^2\theta} &= \frac{\cot\theta\sec\theta}{-\cot^2\theta}\\ &= -\frac{\sec\theta}{\cot\theta}\\ &= -\tan\theta\sec\theta\end{aligned}$$

39. $\dfrac{\cos\left(\dfrac{\pi}{2}-\theta\right)-1}{1+\sin(-\theta)}$

SOLUTION:

$$\begin{aligned}\frac{\cos\left(\dfrac{\pi}{2}-\theta\right)-1}{1+\sin(-\theta)} &= \frac{\sin\theta-1}{1-\sin\theta}\\ &= \frac{-(1-\sin\theta)}{(1-\sin\theta)}\\ &= -1\end{aligned}$$

40. $\dfrac{\sec\theta\sin\theta+\cos\left(\dfrac{\pi}{2}-\theta\right)}{1+\sec\theta}$

SOLUTION:

$$\begin{aligned}\frac{\sec\theta\sin\theta+\cos\left(\dfrac{\pi}{2}-\theta\right)}{1+\sec\theta} &= \frac{\sec\theta\sin\theta+\sin\theta}{1+\sec\theta}\\ &= \frac{\sin\theta(1+\sec\theta)}{1+\sec\theta}\\ &= \sin\theta\end{aligned}$$

41. $\dfrac{\cot\theta\cos\theta}{\tan(-\theta)\sin\left(\dfrac{\pi}{2}-\theta\right)}$

SOLUTION:

$$\begin{aligned}\frac{\cot\theta\cos\theta}{\tan(-\theta)\sin\left(\dfrac{\pi}{2}-\theta\right)} &= \frac{\cot\theta\cos\theta}{-\tan\theta\cos\theta}\\ &= -\cot^2\theta\end{aligned}$$

42. **CCSS CRITIQUE** Clyde and Rosalina are debating whether an equation from their homework assignment is an identity. Clyde says that since he has tried ten specific values for the variable and all of them worked, it must be an identity. Rosalina argues that specific values could only be used as counterexamples to prove that an equation is not an identity. Is either of them correct? Explain your reasoning.

SOLUTION:

Rosalina; there may be other values for which the equation is not true.

43. **CHALLENGE** Find a counterexample to show that $1 - \sin x = \cos x$ is *not* an identity.

SOLUTION:

Sample answer: $x = 45°$

44. **REASONING** Demonstrate how the formula about illuminance from the beginning of the lesson can be rewritten to show that $\cos\theta = \frac{ER^2}{I}$.

SOLUTION:

$$\sec\theta = \frac{I}{ER^2}$$

$$\frac{1}{\cos\theta} = \frac{I}{ER^2} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$ER^2 = I\cos\theta \qquad \text{Crossmultiply}$$

$$\cos\theta = \frac{ER^2}{I} \qquad \text{Divide each side by } I$$

45. **WRITING IN MATH** Pythagoras is most famous for the Pythagorean Theorem. The identity $\cos^2\theta + \sin^2\theta = 1$ is an example of a Pythagorean identity. Why do you think that this identity is classified in this way?

SOLUTION:

Sample answer: The functions $\cos\theta$ and $\sin\theta$ can be though of as the lengths of the legs of a right triangle, and the number 1 can be thought of as the measure of the corresponding hypotenuse.

46. **PROOF** Prove that $\tan(-a) = -\tan a$ by using the quotient and negative angle identities.

SOLUTION:

$$\begin{aligned} \tan(-A) &= \frac{\sin(-A)}{\cos(-A)} \\ &= \frac{-\sin A}{\cos A} \\ &= -\tan A \end{aligned}$$

47. **OPEN ENDED** Write two expressions that are equivalent to $\tan\theta\sin\theta$.

SOLUTION:

Sample answer: $\frac{\sin\theta}{\cos\theta}\cdot\sin\theta$ and $\frac{\sin^2\theta}{\cos\theta}$

48. **REASONING** Explain how you can use division to rewrite $\sin^2\theta + \cos^2\theta = 1$ as $1 + \cot^2\theta = \csc^2\theta$.

SOLUTION:

Divide all of the terms by $\sin^2\theta$.

49. **CHALLENGE** Find $\cot\theta$ if $\sin\theta = \frac{3}{5}$ and $90° \le \theta < 180°$.

SOLUTION:

$$\sin\theta = \frac{3}{5}$$
$$\csc\theta = \frac{5}{3}$$

$$\begin{aligned}1 + \cot^2\theta &= \csc^2\theta\\ 1 + \cot^2\theta &= \left(\frac{5}{3}\right)^2\\ \cot^2\theta &= \frac{16}{9}\\ \cot\theta &= \pm\frac{4}{3}\end{aligned}$$

Since θ is in the second quadrant, cot θ is negative.

Therefore, $\cot\theta = -\frac{4}{3}$.

50. **ERROR ANALYSIS** Jordan and Ebony are simplifying $\frac{\sin^2\theta}{\cos^2\theta + \sin^2\theta}$. Is either of them correct? Explain your reasoning.

Jordan

$$\begin{aligned}\frac{\sin^2\theta}{\cos^2\theta + \sin^2\theta} &= \frac{\sin^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\sin^2\theta}\\ &= \tan^2\theta + 1\\ &= \sec^2\theta\end{aligned}$$

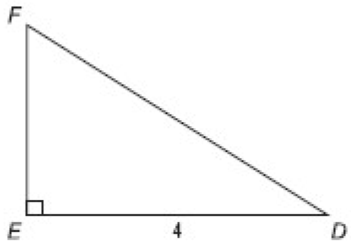
Ebony

$$\begin{aligned}\frac{\sin^2\theta}{\cos^2\theta + \sin^2\theta} &= \frac{\sin^2\theta}{1}\\ &= \sin^2\theta\end{aligned}$$

SOLUTION:

Ebony; Jordan did not use the identity that $\sin^2\theta + \cos^2\theta = 1$ and made an error adding rational expressions.

51. Refer to the figure below. If $\cos D = 0.8$, what is the length of $\overline{DF}$?



A 5
B 4
C 3.2
D $\frac{4}{5}$

SOLUTION:

$$\cos D = \frac{DE}{DF}$$
$$0.8 = \frac{4}{DF}$$
$$DF = 5$$

Option A is the correct answer.

52. **PROBABILITY** There are 16 green marbles, 2 red marbles, and 6 yellow marbles in a jar. How many yellow marbles need to be added to the jar in order to double the probability of selecting a yellow marble?
F 4
G 6
H 8
J 12

SOLUTION:

The probability of getting a yellow marble is $\frac{6}{24}$ or $\frac{1}{4}$.

To double the probability of selecting a yellow marble, we need to add *x* marbles.
That is:

$$\frac{6+x}{24+x} = 2 \cdot \frac{1}{4}$$
$$\frac{6+x}{24+x} = \frac{1}{2}$$
$$12x + 2x = 24 + x$$
$$x = 12$$

Option J is the correct answer.

53. **SAT/ACT** Ella is 6 years younger than Amanda. Zoe is twice as old as Amanda. The total of their ages is 54. Which equation can be used to find Amanda's age?

A $x+(x-6)+2(x-6)=54$
B $x-6x+(x+2)=54$
C $x-6+2x=54$
D $x+(x-6)+2x=54$
E $2(x+6)+(x+6)+x=54$

SOLUTION:
Let x be the age of Amanda.
Therefore, the ages of Ella, Amanda and Zoe are $x-6$, x, $2x$.
Given: $x+x-6+2x=54$.
Option D is the correct answer.

54. Which of the following functions represents exponential growth?

F $y=(0.3)^x$
G $y=(1.3)^x$
H. $y=x^3$
J $y=x^{\frac{1}{3}}$

SOLUTION:
To be an exponential growth, the value in the parenthesis must be greater than one. The variable will be in the exponent.
Therefore, option G is the correct answer.

Find each value. Write angle measures in radians. Round to the nearest hundredth.

55. $\text{Cos}^{-1}\left(-\frac{1}{2}\right)$

SOLUTION:
Use a calculator.

KEYSTROKES: 2nd [COS^{-1}] (–) 1 ÷ 2) ENTER .

$\text{Cos}^{-1}\left(-\frac{1}{2}\right)=2.09$

56. $\text{Sin}^{-1}\frac{\pi}{2}$

SOLUTION:
Use a calculator.

KEYSTROKES: 2nd [SIN^{-1}] 2nd [π] ÷ 2) ENTER .

$\text{Sin}^{-1}\left(\frac{\pi}{2}\right)=$ does not exist

57. $\text{Arctan } \frac{\sqrt{3}}{3}$

SOLUTION:
Use a calculator.

KEYSTROKES: 2nd [TAN^{-1}] 2nd [$\sqrt{\ }$] 3) ÷ 3) ENTER .

$\text{Arctan}\frac{\sqrt{3}}{3} = 0.52$

58. $\tan\left(\text{Cos}^{-1}\frac{6}{7}\right)$

SOLUTION:
Use a calculator.

KEYSTROKES: TAN 2nd [COS^{-1}] 6 ÷ 7) ENTER .

$\tan\left(\text{Cos}^{-1}\frac{6}{7}\right) = 0.60$

59. $\sin\left(\text{Arctan}\frac{\sqrt{3}}{3}\right)$

SOLUTION:
Use a calculator.

KEYSTROKES: SIN 2nd [TAN^{-1}] 2nd [$\sqrt{\ }$] 3) ÷ 3) ENTER .

$\sin\left(\tan^{-1}\frac{\sqrt{3}}{3}\right) = 0.5$

60. $\cos\left(\text{Arcsin}\frac{3}{5}\right)$

SOLUTION:
Use a calculator.

KEYSTROKES: COS 2nd [SIN^{-1}] 3 ÷ 5) ENTER .

$\cos\left(\text{Arcsin}\frac{3}{5}\right) = 0.8$

61. **PHYSICS** The weight is attached to a spring and suspended from the ceiling. At equilibrium, the weight is located 4 feet above the floor. The weight is pulled down 1 foot and released. Write the equation for the distance d of the weight above the floor as a function of the time t seconds assuming that the weight returns to its lowest position every 4 seconds.

SOLUTION:

An equation for the function is $d = a\cos b(t-h)+k$.
At equilibrium, the weight is 4 inches above the floor. Therefore, the vertical shift is k = 4.
The weight is 1 foot closer to the floor at its lowest point, so the amplitude a is 1.
The weight returns to its lowest position every 4 seconds, therefore the period is 4.

$$4 = \frac{2\pi}{|b|}$$
$$b = \frac{\pi}{2}$$

There is no horizontal shift.

So, $d = 4 - \cos\frac{\pi}{2}t$ or $d = 4 - \cos 90°t$.

Evaluate the sum of each geometric series.

62. $\sum_{k-1}^{5}\frac{1}{4}\cdot 2^{k-1}$

SOLUTION:

There are 5 – 1 + 1 or 5 terms.

$$a_1 = \frac{1}{4}\cdot 2^{1-1}$$
$$= \frac{1}{4}$$

Find the sum.

$$S_n = \frac{a_1 - a_1 r^n}{1-r}$$
$$S_5 = \frac{\frac{1}{4} - \frac{1}{4}\cdot 2^5}{1-2}$$
$$= \frac{31}{4}$$

63. $\sum_{k-1}^{7} 81\left(\frac{1}{3}\right)^{k-1}$

SOLUTION:

There are 7 – 1 + 1 or 7 terms.

$$\begin{aligned} a_1 &= 81 \cdot \left(\frac{1}{3}\right)^{1-1} \\ &= 81 \end{aligned}$$

Find the sum.

$$\begin{aligned} S_n &= \frac{a_1 - a_1 r^n}{1-r} \\ S_7 &= \frac{81 - 81 \cdot \left(\frac{1}{3}\right)^7}{1 - \frac{1}{3}} \\ &= \frac{81 - \frac{1}{27}}{\frac{2}{3}} \\ &= \frac{1093}{9} \end{aligned}$$

64. $\sum_{k-1}^{8} \frac{1}{3} \cdot 5^{k-1}$

SOLUTION:

There are 8 – 1 + 1 or 8 terms.

$$\begin{aligned} a_1 &= \frac{1}{3} \cdot (5)^{1-1} \\ &= \frac{1}{3} \end{aligned}$$

Find the sum.

$$\begin{aligned} S_n &= \frac{a_1 - a_1 r^n}{1-r} \\ S_8 &= \frac{\frac{1}{3} - \frac{1}{3} \cdot 5^8}{1-5} \\ &= 32552 \end{aligned}$$

Solve each equation.

65. $a+1=\frac{6}{a}$

SOLUTION:

$$\begin{aligned} a+1&=\frac{6}{a} \\ a^2+a&=6 \\ a^2+a-6&=0 \\ (a+3)(a-2)&=0 \\ a&=2 \text{ or } -3 \end{aligned}$$

66. $\frac{9}{t-3}=\frac{t-4}{t-3}+\frac{1}{4}$

SOLUTION:

$$\begin{aligned} \frac{9}{t-3}&=\frac{t-4}{t-3}+\frac{1}{4} \\ \frac{9}{t-3}-\frac{t-4}{t-3}&=\frac{1}{4} \\ \frac{13-t}{t-3}&=\frac{1}{4} \\ 52-4t&=t-3 \\ 5t&=55 \\ t&=11 \end{aligned}$$

67. $\frac{5}{x+1}-\frac{1}{3}=\frac{x+2}{x+1}$

SOLUTION:

$$\begin{aligned} \frac{5}{x+1}-\frac{1}{3}&=\frac{x+2}{x+1} \\ \frac{5}{x+1}-\frac{x+2}{x+1}&=\frac{1}{3} \\ \frac{-x+3}{x+1}&=\frac{1}{3} \\ 9-3x&=x+1 \\ 4x&=8 \\ x&=2 \end{aligned}$$